Practical Algorithm for Scheduling a Public Transport Line with Integer Solution

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Abstract— Scheduling of a single transit line have been well established and described in the literature in the field. However, the expert community has given little attention to a problem which looks trivial at first sight. Namely, no matter what scheduling procedure or optimization model has been applied, the solutions for the three basic variables that define the transit line timetable: number of vehicles in operation, headway and cycle time, are usually decimal numbers that must be rounded off, while still keeping the basic relation between them. This operation in some cases may cause unfeasible or undesirable line operation. The problem can be defined and solved as nonlinear optimization problem with linear constraints. Due to the relatively limited space of feasible solutions, a rather simple and easy to use algorithm can be developed. It can be used as an independent scheduling procedure or as the last phase of any scheduling model.

Index Terms— Integer solution, optimal algorithm, practical scheduling, scheduling public transport line,
Minimum 7 keywords are mandatory, Keywords should closely reflect the topic and should optimally characterize the paper. Use about four key words or phrases in alphabetical order, separated by commas.

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1 Introduction

Development of optimal service timetables is by far the most important task for every public transport operator. The timetables must provide adequate service to the public, but also the cost efficiency criteria must be considered and ensured. This is not an easy task since the users require good quality of service of which a very important component is the provision of service availability in space and time (good space coverage and frequent service), while the operators are interested in better utilization of the capacities of vehicles in service and lower operating costs. In general, these somewhat conflicting requirements have been resolved by application of optimization methods that include the interests of both: the users and the service providers. The users' requirements have been usually accounted for by means of different measures of level of service, while the operators' interests have been considered trough variables such as operating costs, line and vehicle capacity utilization, etc.

The procedures for computation of transit line operational elements have been well established and described in the literature in the field such as in [1] and [2].

The three basic variables that define the public transport line timetable: the number of vehicles in operation - N, the line headway - h and the line cycle time – T have been usually determined from the basic requirement: the supply should be equal to the demand at given level of service or

Demand = Supply (at given level of service) (1)

The demand for travel by public transport has been expressed by the number of passengers per hour that travel at the maximum load section (MLS) of the line - P_d

The supply has been presented by the line capacity C given in terms of number of provided spaces on the line per hour.

The line capacity may be computed as:

$$C = C_v \cdot f = C_v \cdot \frac{60}{h} \left[\frac{\text{spaces}}{\text{hour}}\right] \tag{2}$$

where:

$C_v \left[\frac{\text{spaces}}{\text{vehicle}}\right]$ - vehicle capacity

$f = \frac{60}{h} \left[\frac{\text{vehicle}}{\text{hour}}\right]$ - line frequency or the number of vehicles past a point on the line during one hour of operation

h [min] - line headway or the time interval between two consecutive vehicles at a point on the line.

The level of public transport service is a multidimensional and complex variable. For the scheduling purpose, the average vehicle occupancy α is usually taken as a measure of the level of service. The average vehicle occupancy shows the average utilization of the vehicle capacity at the MLS and if expressed in percents, it shows what percentage of the available spaces (seating and standing) have been occupied by passengers at most loaded section of the line.

Given the above definitions, the equation (1) becomes:

$$P_d = \alpha \cdot C_v \cdot \frac{60}{h} \tag{3}$$

or, the headway of service can be computed as follows:

$$h = \frac{60 \cdot \alpha \cdot C_v}{P_d} \tag{4}$$

The number of vehicles in operation on a public transport line where a headway h is to be provided, equals to the ratio of the line cycle time T and the line headway h, or:

$$N = \frac{T}{h} \tag{5}$$

Therefore, the scheduling variables N and h can be computed from equations (4) and (5). The cycle time T is known for a specific public transport line with defined route.

This scheduling method is simple enough and it is quite

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easy for use. However, the expert community has given very little attention to a problem which looks trivial at first sight, but nevertheless is a problem that operators have been faced with in their efforts to optimize the operation of a transit line. This problem arises from the fact that the three basic variables that define the transit line timetable: N, h and T, ought to be integer numbers, while the solutions of the equations (4) and (5) are usually decimal numbers.

The number of vehicles in operation obviously must be an integer number, while the operators would not like to make and publish timetables, in which the departure and arrival times are shown in decimal numbers. In addition, these variables must satisfy the equation (5). As a consequence, during the scheduling procedure, these variables must be rounded off and their values must be adapted to satisfy the relationship (5). In many cases, these computing procedures (the rounding off and the fitting to the equation 5) will not cause any problems, but in some cases, they may lead to an integer solution that violates the initial conditions or leads to less efficient line operation.

In this article, the author has made an effort to present a solution to this problem by defining a scheduling procedure of a public transport line that will give integer solution for selected optimization criteria.

2. The Nature of the Problem

Depending on the values of the input variables, the scheduling procedure described above, may lead to bad solution in two cases:

Case 1: Problem of lower or greater average vehicle occupancy α than the one desired or initially defined

From the equation (4) α can be expressed as:

$$\alpha = \frac{P_d}{60 \cdot C_v} \cdot h \tag{6}$$

If h is the decimal solution from equation (4), and h_r is the integer number obtained by rounding off h, then the difference $\Delta h = h - h_r$ may take values from 0 to ±0,5.

Therefore, the true value of α after rounding off h will become:

$$\alpha_{true} = \frac{P_d}{60 \cdot C_v} \cdot h_r = \frac{P_d}{60 \cdot C_v} \cdot (h \pm \Delta h) = \alpha \pm \frac{P_d}{60 \cdot C_v} \cdot \Delta h \tag{7}$$

For example, in case of a bus line service with standard buses (80 spaces) and the passenger design volume of 2880 passengers per hour, and the pre-defined average vehicle occupancy of 0,9 (90% of the vehicle capacity) the maximum deviation of α occurs for $\Delta h = \pm 0{,}5$ and the true α in such case will become:

$$\alpha_{true} = \alpha \pm \frac{P_d}{60 \cdot C_v} \cdot \Delta h = 0{,}9 \pm 0{,}6 \cdot 0{,}5 = 0{,}9 \pm 0{,}3 \tag{8}$$

The result from equation (8) means that the operator may end up scheduling the line operation with unacceptable vehicle overloading ($\alpha = 1{,}2$) or will operate the line with lower than desired efficiency ($\alpha = 0{,}6$) compared to the initially defined $\alpha = 0{,}9$.

Case 2: Problem of shorter or much longer cycle time than the one that is determined for a specific public transport line. The shorter cycle time is infeasible solution since the vehicles cannot move faster than the minimum cycle time on a defined route. The longer cycle time means less efficient operation, since the added time will be spent as an unproductive time - standing at line terminals.

If N is the decimal solution from equation (5), and N' is the integer number obtained by rounding off N, then the difference $\Delta N = N - N'$ may take values from 0 to ±0,5.

As a result, the true value of T after rounding off N becomes:

$$T' = N' \cdot h = (N \pm \Delta N) \cdot h = T \pm \Delta N \cdot h \tag{9}$$

The greatest deviation occurs for $\Delta N = \pm 0{,}5$. In case of operation with long headways (for example 30 minutes), the final value for the cycle time will be 15 minutes lower or greater than the initially determined cycle time.

3. Review of the literature on the subject

The review of the previous work on this subject has shown a variety of models developed for scheduling a single or a network of transit lines. Majority of this models focus on optimization of the operation of a transit line or a network, with no regard on the problem of integer solution for the main scheduling variables. Only a few, take into consideration the integer solution problem trough application of mathematical programing models with mixed or integer variables.

Lampkin and Saalmans [3] proposed a constrained optimization problem for determination of the line frequency. The objective was to minimize the total travel time for a given fleet size constraint and a random search procedure was used for solution.

Rea's [4] model search for an optimum bus network by adjusting iteratively the frequencies and type of buses.

Silman et al. [5] determined optimum frequencies for a set of bus routes and fleet size, which could minimize the total travel time and discomfort by a gradient method procedure.

Hsu and Surti [6] used the concept of marginal ridership to determine adequate frequency for each route for a given fleet size.

Scheele [7] proposed a mathematical programming algorithm. The problem of optimal bus frequencies is solved by a gradient projection method.

Dubois et al. [8] used a two-step procedure for frequency determination.

Furth and Wilson [9] model allocates the available buses between time period and between routes so as to maximize net social benefit subject to constraints on total subsidy, fleet size and level of vehicle loading.

Han and Wilson [10] model recognizes passenger route choice behaviour and seeks to minimize a function of passenger wait time and bus crowding subject to constraints on number of buses available and the provision of enough capacity on each route.

Shih and Mahmassani [11] have also used the concepts of optimal vehicle size, frequency adjustment for co-ordinated routes, timed transfer and transit centers.

Dashora [12] used an expert-system based model which allocates the buses to different routes between a maximum and minimum number, based on additional bus allocation factor (saving in waiting time/additional cost of operation) criteria.

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The candidate route for increasing a bus is the one for which additional bus allocation factor is maximum. The buses are allocated till fleet size is exhausted.

Kidwai et al. proposed two stage procedure for bus scheduling with the main objective to determine the total bus fleet size for a specific urban area to be served [13]. In the first stage, they estimate the transit line frequencies and number of vehicles in operation at individual line level. This is done by considering the number of passengers at the maximum load section and the vehicle capacity. Within the first stage, they proposed rounding off the results for the number of vehicles and vehicle frequencies, but they do not consider the consequences of that mathematical operation. In the second stage they use the genetic algorithm in order to rationalize the total number of vehicle at the public transport network level.

Some researchers have tried optimization techniques based on the biological evolution. The genetic algorithm and artificial neural network, for transit network design has been proposed by Xiong and Schneider [14], Chakraborty et. al. [15] Pattnaik et. al. [16] Chien et. al. [17], Bielli et. al. [18], Tom and Mohan [19] and Ngamchai and Lovell [20] among the others.

4. Algorithm for optimal scheduling of a transit line

For the purpose of scheduling a public transport line, the operator can define the acceptable level of service by defining the range of acceptable values for α that is $\alpha_{\min}$ *to* $\alpha_{\max}$.

In addition, the operator can define the acceptable values for the cycle time $T_{\min}$ *to* $T_{\max}$.

Since the line cycle time can be expressed as a sum of the terminal to terminal travel times plus the standing times at terminals

$$T = t_v^{AB} + t_v^{BA} + t_t^{A} + t_t^{B} \quad (10)$$

where

t_v^{AB}, t_v^{BA} are the terminal to terminal travel times in direction AB and BA

t_t^{A}, t_t^{B} are the standing times at terminal A and B

the acceptable minimum and maximum cycle times are determined by the acceptable minimum and maximum terminal times.

Given the equation (4), the range of acceptable values for α will result in range of acceptable integer values for the line headway:

$$h_{\min} = \frac{60 \cdot \alpha_{\min} \cdot C_v}{P_d} \quad \text{round up} \quad (11)$$

$$h_{\max} = \frac{60 \cdot \alpha_{\max} \cdot C_v}{P_d} \quad \text{round down} \quad (12)$$

Finally, from equation (5), given the range of acceptable values for h and T, the range of acceptable integer values for N can be computed.

$$N_{\min} = \frac{P_d \cdot T_{\min}}{60 \cdot C_v \cdot \alpha_{\max}} \quad \text{round up} \quad (13)$$

$$N_{\max} = \frac{P_d \cdot T_{\max}}{60 \cdot C_v \cdot \alpha_{\min}} \quad \text{round down} \quad (14)$$

Now, solution to the problem can be defined as one that minimizes the number of vehicle in operation for a given passenger demand at desired level of service or

Minimize the function

$$\min \; N = \frac{T}{h} \quad (15)$$

in such a way that the solution for the variables N, h, and T satisfy the following constraints

$$\begin{aligned} h_{\min} &\le h \le h_{\max} \\ N_{\min} &\le N \le N_{\max} \\ T_{\min} &\le T \le T_{\max} \end{aligned} \quad (16)$$

and N, h and T are positive integer numbers (N, h, T $\in \mathbb{Z}+$)

This is a nonlinear integer optimization problem with linear constraints. A solution of this problem will provide a public transport line operation with minimum number of vehicles, for given passenger demand and predefined level of service. In case of multiple solutions, the one with the lowest headway is taken as a best solution.

The defined nonlinear integer problem can be solved by means of branch and bound method [21]. However, this method is quite cumbersome. Given the simplicity of the optimization function and the constraints, as well as the fact that the space of feasible solutions is rather limited to a number of positive integer values, a much faster and simpler solution can be derived.

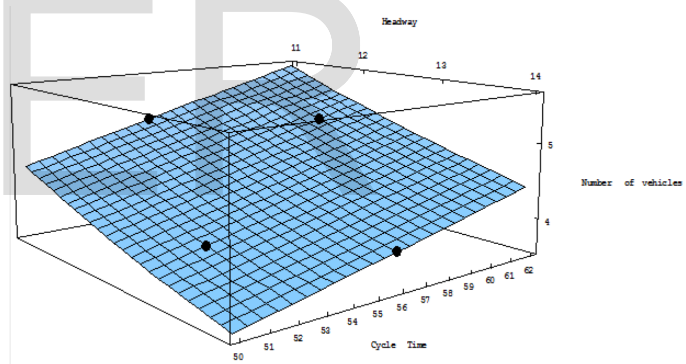

Fig.1 The function $N = \frac{T}{h}$, the space defined by the constraints and the possible integer solutions

For predefined acceptable values for the cycle time of a specific public transport line $T_{\min} \le T \le T_{\max}$ and the desirable level of service defined by the acceptable values for coefficient α ($\alpha_{\min} \le \alpha \le \alpha_{\max}$), the space of acceptable integer solutions can defined as given in equation (16).

Since we are looking for an integer solution that involves minimum possible number of vehicles in operation for a given level of service, the search can start along a linear function

$$T = N_{\min} \cdot h \quad (17)$$

It must be checked if in the interval $h_{\min} \le h \le h_{\max}$ there exist integer number T that satisfies the requirement $T_{\min} \le T \le T_{\max}$. If such integer value exists, then the solution is reached.

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The search can start for $h = h_{\min}$.

In general, for $N = N_{\min}$ and $h = h_{\min}$ it is true that $N_{\min} \cdot h_{\min} < T_{\min}$ since

$$N_{\min} \cdot h_{\min} = \frac{P_d \cdot T_{\min}}{60 \cdot \alpha_{\max} \cdot C_v} \cdot \frac{60 \cdot \alpha_{\min} \cdot C_v}{P_d} = \frac{\alpha_{\min}}{\alpha_{\max}} \cdot T_{\min} \quad (18)$$

However, since $h_{\min}$ and $N_{\min}$ are integer numbers that are computed from equations (11) and (13) by rounding up, it is possible to get $N_{\min} \cdot h_{\min} \geq T_{\min}$. In this case, the $N_{\min} \cdot h_{\min} = T$ is a solution to the problem.

If for $N_{\min}$ and $h_{\min}$ the condition $T_{\min} \leq T \leq T_{\max}$ is not satisfied, then it can be checked, what integer value of h can give a solution $N_{\min} \cdot h \geq T_{\min}$

$$h = \frac{T_{\min}}{N_{\min}} \text{ round up} \quad (18)$$

If $h_{\min} \leq h \leq h_{\max}$ then the solution is reached. If not, that is if $h > h_{\max}$ then obviously there is no solution for $N = N_{\min}$.

In this case the lowest value for N can be found for $h = h_{\max}$ and the solution can be determined for

$$N = \frac{T_{\min}}{h_{\max}} \text{ round up} \quad (19)$$

Now, the final solution to the problem will be the set of integer values N, $h_{\max}$ and $T = N \cdot h_{\max}$.

The entire procedure can be defined by means of pretty simple algorithm given in a form of flowchart in the Appendix 1.

5. An Example Case

Public transport line 5 in Skopje (Novo Lisice – Deksion) is served by double-decker buses. This line has the following data needed for its scheduling:

$C_v = 75\left[\frac{\text{spaces}}{\text{vehicle}}\right]$	vehicle capacity
$0{,}75 \leq \alpha \leq 0{,}92$	acceptable range of vehicle occupancy rate at MLS
$5 \leq t_t \leq 12$ [min]	acceptable values for terminal times

Minimum acceptable cycle time:

$$T_{\min} = t_v^{AB} + t_v^{BA} + t_{t,\min}^{A} + t_{t,\min}^{B} = 50 + 50 + 2 \cdot 5 = 110\,[\text{min}]$$

Maximum acceptable cycle time:

$$T_{\max} = t_v^{AB} + t_v^{BA} + t_{t,\max}^{A} + t_{t,\max}^{B} = 50 + 50 + 2 \cdot 12 = 124\,[\text{min}]$$

$P_d = 590\left[\frac{\text{passengers}}{h}\right]$ number of passengers at MLS

Step 1: Determination of the feasible space of solutions

$$h_{\min} = \frac{60 \cdot \alpha_{\min} \cdot C_v}{P_d} = \frac{60 \cdot 0{,}75 \cdot 75}{590} = 5{,}72 \approx 6\,[\text{min}] \text{ (round up)}$$

$$h_{\max} = \frac{60 \cdot \alpha_{\max} \cdot C_v}{P_d} = \frac{60 \cdot 0{,}92 \cdot 75}{590} = 7{,}02 \approx 7\,[\text{min}] \text{ (round down)}$$

$$N_{\min} = \frac{P_d \cdot T_{\min}}{60 \cdot C_v \cdot \alpha_{\max}} = \frac{590 \cdot 110}{60 \cdot 75 \cdot 0{,}75} = 15{,}67 \approx 16\,[\text{vehicles}] \text{ (round up)}$$

$$N_{\max} = \frac{P_d \cdot T_{\max}}{60 \cdot C_v \cdot \alpha_{\min}} = \frac{590 \cdot 124}{60 \cdot 75 \cdot 0{,}92} = 21{,}68 \approx 21\,[\text{vehicles}] \text{ Round down)}$$

Step 2:

N = N_{min} = 16 vehicles
h = h_{min} = 6 minutes
T = N h = 16 x 6 = 96 minutes

Checking if $T_{\min} \leq T \leq T_{\max}$

$T < T_{\min}$ the condition is not satisfied

Step 3:

N = N_{min} = 16 vehicles

$$h = \frac{T_{\min}}{N} = \frac{110}{16} = 6{,}87 \approx 7 \text{ minutes (round up)}$$

T = N h = 16 x 7 = 112 minutes

Checking if $T_{\min} \leq T \leq T_{\max}$

110 < 112 < 124 (OK – solution)

Final integer solution that satisfy the initial conditions:

N = 16 vehicles
h = 7 minutes
T = 112 minutes

Checking the level of service indicator for the adopted schedule parameters:

$$\alpha = \frac{P_d \cdot T}{60 \cdot C_v \cdot N} = \frac{590 \cdot 112}{60 \cdot 75 \cdot 16} = 0{,}917 < 0{,}92$$

6. Conclusion

Regardless of the scheduling optimization model that is used for development of public transport line timetables, the operators are faced with the need to round off the results for the three major timetable variables: number of vehicles in operation, line cycle time and the line headway. This rounding off may cause ending up with violation of initial conditions regarding the desired values for the level of service or the line cycle times.

The search for a solution of this problem results in solving nonlinear integer optimization programming with linear constraints. In general, this is not an easy task, but the simplicity of the optimization function and the relatively small space of feasible solutions, allows a development of fairly simple algorithm that allows rather fast and easy solution of the problem.

The presented approach has a number of advantages:

- The proposed algorithm is easy to use
- It can be used as a final phase with any of the existing different models for optimization of a single transit line operation or optimization of the entire transit network, or it can be used as independent optimization procedure
- The proposed algorithm can be easily programmed using any of the computer programme languages
- Similar approach can be developed for different optimization criteria

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APPENDIX 1

ALGORITHM FOR OPTIMAL SCHEDULING OF PUBLIC TRANSPORT LINE

Integer solution for minimum number of vehicles in operation at given level of service